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Confinement on the Brane

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Abstract

A non-perturbative confinement mechanism has been proposed to explain the fate of the unbroken gauge group on the world-volume of annihilating D-brane-anti-D-brane pairs. In this paper, we examine this phenomenon closely from several different perspectives. Existence of the confinement mechanism is most easily seen by noticing that the fundamental string emerges as the confined electric flux string at the end of the annihilation process. After reviewing the confinement proposal in general, this is shown explicitly in the D2-anti-D2 case in the M-theory limit. Finally, we address the crucial issue of whether and how confinement occurs in the weakly coupled limit of string theory.

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1 Introduction

D-branes and anti-D-branes are believed to undergo an annihilation process analogous to that of ordinary particles and anti-particles. However, unlike ordinary particles, D-branes carry local degrees of freedom associated with open strings. An obvious question is what happens to these degrees of freedom when the branes and anti-branes annihilate. Given that we anticipate a supersymmetric vacuum without the branes, it is clear that all such open string modes must be removed from the spectrum of physical states, leaving behind a pure closed string theory.¹ One would like to understand this process from the open string point of view.

In the weak string coupling regime, and at energies well below the string scale ($\alpha'^{-1/2}$), D-brane dynamics can be approximated by concentrating on the lowest lying modes of the open strings, and ignoring the coupling to bulk closed strings. The result is a world-volume field theory [1, 2], which includes, among other fields, world-volume gauge fields. In the brane-anti-brane system the world-volume theory contains one gauge field on the brane and another on the anti-brane, as well as a complex tachyonic scalar from the brane-anti-brane string. Both gauge fields must be removed from the spectrum in the annihilation process. The question we would like to ask is whether and how this process can be effectively described at the level of the world-volume gauge theory.

We can get some insight into this question by considering the possible by-products of such an annihilation. It is well-known that a non-vanishing world-volume gauge field strength serves as a source for various space-time fields [3, 4]. Once the D-brane world-volume disappears, these fields must be supported by remnant space-time objects. For example, magnetic flux on a Dp -brane is a source for the R-R $(p-1)$ -form field, and therefore [5] induces a $D(p-2)$ -brane charge. Charge conservation therefore implies that after the annihilation one is left with precisely such a D-brane. This requires, however, that there exists a *localized* magnetic vortex solution in the world-volume theory, which suggests that the annihilation process incorporates a Higgs mechanism.²

This is precisely what happens. The tachyon plays the role of the Higgs field, and the Higgs mechanism proceeds via tachyon condensation [7, 8]. This process induces a mass for a linear combination of the gauge fields, thereby removing it from the low-energy spectrum. However, the other combination, under which the tachyon is neutral, remains unbroken and appears to stay massless [9, 10]. This is the puzzle of the unbroken $U(1)$.

¹Up to lower dimensional branes as by-products of the annihilation process.

²This argument is due to Kimyeong Lee [6].

To understand what happens to the unbroken $U(1)$, we consider this time electric flux on the Dp -brane (on a circle), which serves as a source for the NS-NS 2-form field. The flux induces a (wound) fundamental string charge [2], so in this case one must be left with a fundamental string after the annihilation. From the world-volume perspective, this requires electric flux to be *confined* to a thin tube, where the confinement scale is set by the tension of the fundamental string. This suggests that the world-volume description of the annihilation process should also incorporate a confinement mechanism. In fact, one of the authors has proposed in [11] that confinement indeed occurs and removes the unbroken $U(1)$ from the low energy dynamics. For $p \geq 3$, it was argued that confinement is driven by a dual Higgs mechanism, involving the condensation of magnetically charged tachyonic states, which are realized as $D(p-2)$ -branes suspended between the Dp -brane and the anti- Dp -brane. However, since this mechanism is non-perturbative it is difficult to establish rigorously at weak string coupling. Furthermore, it does not seem to be applicable directly to the cases with $p < 3$. This paper will, in part, address these difficulties.

More recently it has been proposed by A. Sen that the removal of the additional gauge sector might be understood at tree level in string perturbation theory [12]. Sen has shown, under the crucial assumption that the minimum of the perturbative tachyon potential precisely cancels the tension of the brane, that the kinetic term for the unbroken gauge field vanishes when the tachyon attains its vacuum expectation values. This in turn implies that charged states are removed from the spectrum, which also suggests a form of confinement, albeit one that is completely perturbative in origin.

We shall argue in section 4 that this observation is actually incomplete by itself to resolve the puzzle of the unbroken $U(1)$. For instance, it cannot explain why the electric flux condenses into fundamental strings. Instead, it will actually help us to realize the non-perturbative world-volume confinement picture of Ref. [11] much more concretely. Specifically, we will use the result of [12] for the $D1$ -anti- $D1$ and $D2$ -anti- $D2$ systems, and derive the tension of the confined electric flux tube, thereby confirming its identification with the fundamental string. In the process, we propose an alternate scenario for the tachyon potential, where the supersymmetric vacuum is restored only when both perturbative and non-perturbative tachyonic directions are turned on.

It is the purpose of this paper to offer additional arguments in favor of the proposal of [11], and study its implication in the weakly coupled limit of string theory. In particular we will clarify the relationship between this non-perturbative confinement mechanism and Sen's recent observations on the perturbative effective action of unstable D-branes. In section 2 we review the proposed confinement mechanism in the brane-anti-brane system,

and propose the analogous mechanism for the unstable D-branes of Type II string theory. In section 3, we describe the D2-anti-D2 case in purely geometric terms using M-theory, and demonstrate explicitly the production of fundamental strings as confined electric flux. We then apply a similar geometrical approach to other systems, and derive additional by-products of brane-anti-brane annihilation. In section 4 we will come back to the all important question of how the (non-perturbative) confinement occurs in the weak coupling limit, and how Sen’s observation fits into this phenomenon.

2 The Confinement Mechanism

The confinement mechanism in question involves non-perturbative objects like open $D(p-2)$ branes; it is rather difficult to probe in the perturbative regime of string theory. Nevertheless, there is ample evidence that such a mechanism should exist in the brane-anti-brane annihilation process, not the least of which is that it would solve the puzzle of the unbroken $U(1)$. One of more compelling pieces of evidence is the charge conservation argument outlined in the introduction. The qualitative dynamics of confinement can be most easily understood as a Higgs mechanism of an antisymmetric tensor field which is dual to the confined gauge field [11]. We will start by reviewing the original proposal in Section 2.1, which holds for Dp -anti- Dp pairs for $p \geq 3$. ($p \leq 2$ cases are special because of the low dimensional nature, and needs a different approach. See Section 3.1 and Section 4.) This will be followed by a discussion of confinement in other cases, such as in multiple brane-anti-brane pairs and unstable D-branes.

2.1 Brane-anti-Brane

The Dp -anti- Dp system includes in its world-volume two gauge fields A and A' on the brane and on the anti-brane, respectively, and a complex tachyonic scalar T from strings ending on both. To determine the charge of T , recall that the world-volume interaction which assigns charge to the endpoint of a string is given by

$$\int *F^{(2)} \wedge B^{(2)} , \quad (2.1)$$

where $B^{(2)}$ is the (pullback of the) NS-NS 2-form field. The endpoints of open strings will thus carry electric charge with respect to the world-volume gauge fields A on Dp and A' on anti- Dp . The relative sign of two charges is a matter of convention, which we will choose by saying that the open string is neutral under the symmetric combination $A_+ = A + A'$. With this convention, the open string carries a unit electric charge with respect to $A_- = A - A'$.

The low-energy world-volume theory is therefore an Abelian Higgs model with a gauge group $U(1) \times U(1)$ broken spontaneously to the diagonal subgroup $U(1)_+$. As the tachyon is expected to condense to a value of the order of the string scale, this generates an $\mathcal{O}(\alpha'^{-1/2})$ mass term for A_- , thereby removing it from the low-energy spectrum. A by-product of this Higgs effect is a magnetic vortex. In an Abelian Higgs model, the topological soliton arises from the winding number of the scalar expectation values at infinity, which induces quantized magnetic flux. This magnetic vortex can be shown to carry quantized $D(p-2)$ -brane charges, owing to the topological term [3],

$$\int F^{(2)} \wedge C^{(p-1)} , \quad (2.2)$$

where $C^{(p-1)}$ is the (pullback of the) R-R $(p-1)$ -form field.

For $p \geq 3$, the world-volume theory also contains non-perturbative magnetically charged objects, corresponding to $D(p-2)$ -branes which are suspended between the brane and the anti-brane [4]. Their charges are determined by the topological term in (2.2), which tells us that the boundaries are magnetic monopoles. Since the two couplings in (2.1) and (2.2) have opposite parities under orientation reversal of the world-volume, the non-perturbative states carry magnetic charge with respect to A_+ and are neutral under A_- [11].

With the exception of the $p = 3$ case, we do not know much more about these magnetic states, since they correspond to extended objects in the world-volume. In particular, their tension should in principle be determined by the ground state energy of the suspended $D(p-2)$ -brane, which we do not know in general. For $p = 3$ the magnetic state is a particle, which corresponds to a suspended D-string. Here too, we cannot in general compute the mass of the state. At large string coupling ($g_s \gg 1$) however, the quantization of the D-string is the same as the quantization of the fundamental string for $g_s \ll 1$. Thus for $g_s \gg 1$ the ground state of this D-string corresponds to a magnetically charged tachyon \tilde{T} . Since this tachyon is charged only under A_+ , its condensation leads to a dual Higgs mechanism, in which the dual gauge field \tilde{A}_+ becomes massive. In the original variables, this translates into confinement. The “solitonic” by-product of confinement is a thin electric flux string, which, due to the coupling in (2.1), carries the fundamental string charge.

For $p > 3$ the situation is more complicated. The question is how to describe higher-dimensional analogs of the dual Higgs mechanism, whereby an extended magnetic state becomes tachyonic (*i.e.* negative tension-squared) and condenses. Generalization of the Higgs mechanism to higher rank anti-symmetric tensor fields is straightforward [11, 14], however, the difficulty lies in describing the magnetically charged tachyonic objects.

On the other hand, it turns out that for the special case of $p = 4$ one can give an indirect argument that there should be such a generalized Higgs mechanism for anti-symmetric tensor fields on branes [11]. The D4-anti-D4 system in Type IIA string theory corresponds to an M5-anti-M5 system wrapped on the circle in the eleventh dimension of M-theory. The ordinary Higgs mechanism in the Type IIA system must therefore lift to an analogous mechanism for the self-dual and anti-self-dual two-forms, which live on the M5-brane and anti-M5-brane, respectively.

This also implies, in particular, that M5-anti-M5 annihilation can produce M2-branes as solitonic by-products. This follows from the fact that, in the Type IIA picture, a transverse M2-brane is a D2-brane, which is realized as a world-volume vortex in the D4-anti-D4 system. On the other hand, a wrapped M2-brane corresponds to a fundamental string in Type IIA, which appears in the D4-anti-D4 system as confined electric flux of A_+ . From the perspective of D4-anti-D4, this confinement proceeds via a dual Higgs mechanism for A_+ , as anticipated. This establishes that for the D4-anti-D4 system the Higgs and confinement phenomena are described by a single, Higgs-like, effect on an M5-anti-M5 pair. So, at least for this system, we see that both mechanisms do occur.

This example touches upon the other crucial question of whether the (perturbative) Higgs mechanism and the (non-perturbative) dual Higgs mechanism occur simultaneously. Naively, one would expect that when one sector is weakly coupled the other is necessarily strongly coupled, due to the duality transformation. This would imply that we can discuss only one of the two sectors reliably, which may cast some doubt on the resolution of the puzzle of the unbroken $U(1)$ by confinement. The above example of D4-anti-D4 alleviates some of this doubt, given that the two phenomena are actually the same effect in M-theory, but it cannot be generalized to other cases. Thankfully, however, it turns out that this problem is naturally resolved. The two mechanisms, for a very interesting reason, can be simultaneously described in weak-coupling descriptions. We will come back to this crucial question in Section 4.

2.2 Multiple Brane-anti-Brane

For a system of N D p -anti-D p pairs the gauge group is $U(N) \times U(N)$, and the “electric” tachyon transforms in the bi-fundamental representation, *i.e.* $(\mathbf{N}, \overline{\mathbf{N}}) \oplus (\overline{\mathbf{N}}, \mathbf{N})$. The candidate for the “magnetic” tachyon corresponds to an open D $(p-2)$ -brane, and transforms in the bi-fundamental representation of the dual gauge group. As the former condenses, the gauge group is broken to the diagonal subgroup $U(N)_+$, and we must somehow explain how the magnetic tachyon confines the remaining non-Abelian gauge

sector.

The argument for a single pair is not applicable here. The main obstacle is the question of how the magnetic tachyon transforms under the dual unbroken gauge group. In fact, it is not clear how to dualize non-Abelian gauge fields to begin with. One way to get around this problem is to break the original non-Abelian group to its Cartan subgroup $U(1)^{2N}$ by turning on the adjoint scalar fields corresponding to separating the different Dp -anti- Dp pairs. In this Coulomb branch, the physics of brane-anti-brane annihilation is then a simple generalization of the single Dp -anti- Dp case. Namely, N of the $U(1)$ factors are broken by the Higgs mechanism, and the others are confined. The N confined $U(1)$ gauge fields belong to the diagonal subgroup $U(N)_+$, so the dual Higgs mechanism confines the gauge charges associated with the N mutually commuting generators of $U(N)_+$. At the origin of the Coulomb branch the $U(N)_+$ symmetry is restored, and the Cartan generators are no longer singled out. This suggests, but does not prove, that the entire non-Abelian gauge sector will be confined.

2.3 Unstable D-branes

We would now like to extend the confinement picture to the unstable D-branes of Type II string theory [15, 16]. These are Dp -branes where p has the “wrong” values, namely p is odd in Type IIA and even in Type IIB. They correspond to boundary states which have components in the NS-NS sector but not in the R-R sector. Consequently, the open string spectrum consists of an unprojected NS sector and an unprojected R sector, and includes a real neutral tachyon as well as a massless $U(1)$ gauge field [15, 16].¹ Here one encounters an apparent puzzle in trying to apply the confinement mechanism for the unbroken gauge group, which in this case is just the $U(1)$. Since the $D(p-2)$ -brane, like the Dp -brane, does not carry R-R charge, it does not appear to give rise to any (magnetic) world-volume charge on the Dp -brane.

The resolution lies in the realization of the unstable Dp -brane of Type IIA(B) as a projection of the Dp -anti- Dp system in Type IIB(A) by the discrete symmetry generated by $(-1)^{F_L}$, where F_L denotes the left-moving part of the space-time fermion number [15]. This follows directly from the action of $(-1)^{F_L}$ on the Chan-Paton factors of the open strings in the Dp -anti- Dp system. In particular, the lowest lying (bosonic) states are given

¹For N coincident unstable D-branes the gauge group is $U(N)$, and the tachyon transforms in the adjoint representation.

by

$$\begin{bmatrix} A & T \\ T^* & A' \end{bmatrix}, \quad (2.3)$$

out of which only the combinations $A_+ = A + A'$ and $T + T^*$ survive the projection. Since the tachyon is neutral under A_+ , these are precisely the lowest-lying degrees of freedom of the unstable D-brane.

We now show that there is indeed a $(p-3)$ -dimensional object in the world-volume that is magnetically charged under the $U(1)$ gauge group. Recall that before the projection, the relevant magnetic state was produced by a (BPS) D $(p-2)$ -brane suspended between the D p -brane and the anti-D p -brane. Since all R-R fields are odd under the operator $(-1)^{F_L}$, D-branes are mapped to anti-D-branes, and vice-versa. In particular, the D $(p-2)$ -brane is mapped to an anti-D $(p-2)$ -brane, but at the same time the D p -brane and anti-D p -brane are exchanged. As a result, the magnetic state actually survives the projection. As shown in the previous section, this state is magnetically charged under A_+ and neutral under A_- . It is therefore magnetically charged under the gauge field on the unstable D-brane. As in the brane-anti-brane system, it can therefore lead to confinement of the gauge field by its condensation.

3 Geometric Confinement

In the previous section we tried to understand the confinement mechanism from the world-volume field theory viewpoint. On the other hand, one crucial aspect of confinement, namely the existence of a confined electric flux string and its identification with the fundamental string, can also be seen from the space-time perspective. As mentioned in the introduction, one merely needs to invoke charge conservation to see that electric flux must condense into fundamental strings. In particular, there are cases where this process can be seen explicitly from purely geometric considerations, which we propose to call “geometric confinement.” These are cases where electric flux can be translated into a winding of branes along a compact circle, such as the M-theory circle along X^{11} .

3.1 D2-anti-D2

Confinement through the dual Higgs effect does not apply directly to the system of D2-anti-D2. One may imagine that a related, Polyakov type mechanism works for this 2+1-dimensional system. This will be addressed in Section 4. Independently of this, however, confinement on D2-anti-D2 can be seen in another way, it turns out. Let us

first recall that a D2-brane is equivalent to an M2-brane that is transverse to the 11-th dimensional circle [17]. The gauge field on D2 is then realized as the dual of the periodic scalar field $\eta \equiv X^{11}/R_{11}$ on the 2+1-dimensional world-volume of the membrane,

$$\partial_i A_j - \partial_j A_i = \epsilon_{ijk} \partial_k \eta, \quad (3.1)$$

where the derivatives are with respect to the three world-volume coordinates x^i . With this in mind, consider an electric flux configuration along the direction of x^1 ,

$$\partial_0 A_1 - \partial_1 A_0 = f(x^2) = \partial_2 \eta. \quad (3.2)$$

Using this relationship to solve for η , we find that the M2-brane is actually winding around the circle along X^{11} ,

$$X^{11}(x^2) = R_{11} \int_{-\infty}^{x^2} f(s) ds. \quad (3.3)$$

Now suppose that we have a D2-anti-D2 pair, with a unit of flux on D2 (so that the corresponding M2 brane winds around the compact circle once) and no flux on anti-D2. We assume that both membranes are asymptotically flat ($X^1 = x^1$ and $X^2 = x^2$). The situation is illustrated in Figure 1.

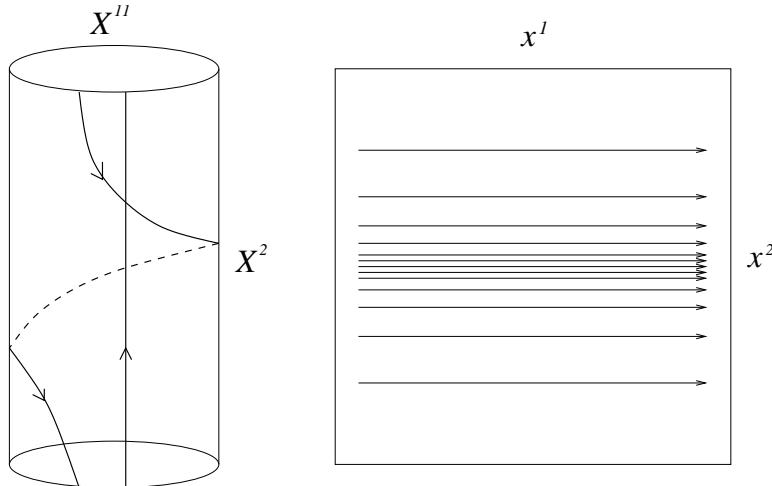


Figure 1: An M2-brane winding around X^{11} once and an anti-M2-brane with no winding (left). On the D2-brane, the winding translates into a unit of electric flux (right).

Now let us ask what happens to this flux once the pair annihilates. The annihilation process dissipates the tension of the branes: the more complete the process, the lower the energy. On the other hand, the winding cannot be removed in this process, and at

the end one finds a remnant which is an M2-brane tightly wrapped around the circular X^{11} direction. Scanning along the x^2 direction, we find that before the annihilation the flux $\partial_2\eta$ is distributed over some finite width along x^2 , while after the annihilation the flux is practically localized at some definite x^2 . From the world-volume perspective, one finds a confined electric flux string. From the space-time viewpoint, this is nothing but the fundamental string.

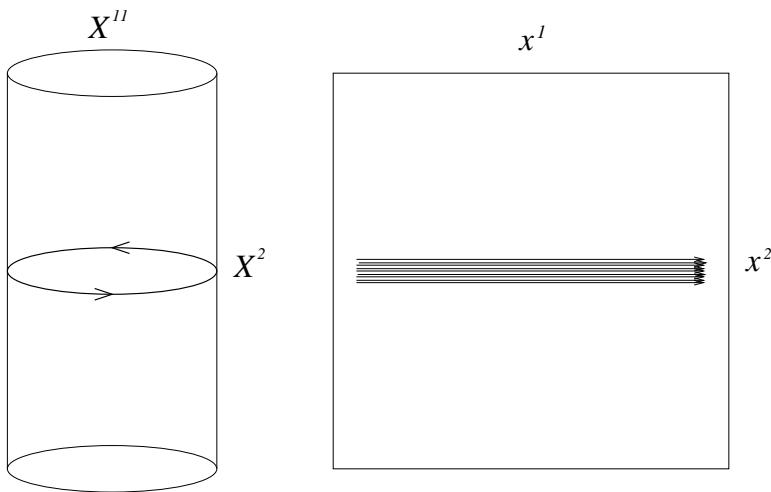


Figure 2: Most of the M2-brane annihilates with the anti-M2-brane, but part of it remains and winds around X^{11} . The result is a single longitudinal M2-brane along X^{11} (left). Seen from the original transverse world-volume of the M2-brane (right), this translates into a tightly confined flux string.

We discovered that certain electric flux is confined upon D2-anti-D2 annihilation, and that the resulting confined string is the fundamental string of Type IIA theory. This shows that confinement of the world-volume gauge field indeed occurs as part of the annihilation process.

One may still ask if this indeed corresponds to confinement of the correct $U(1)$ gauge field. Recall that one combination of $U(1)$'s should be actually Higgsed. To see this clearly, let us recall the fact that anti-branes are nothing but branes with opposite orientation. This distinction shows up in Hodge-dual operations one performs to convert the periodic scalar to a vector field. If we denote the periodic scalar and its dual gauge field on the anti-M2-brane by η' and A' , we find

$$\partial_i A'_j - \partial_j A'_i = -\epsilon_{ijk} \partial_k \eta'. \quad (3.4)$$

Note the sign on the right hand side. The net relative winding lies in the linear combination $\eta - \eta'$, so the confined gauge field is the linear combination $A_+ = A + A'$.

On the other hand, from the decomposition of the 11-dimensional metric to the 10-dimensional metric plus dilaton and R-R 1-form gauge field $\mathcal{C}^{(1)}$, we learn that the sum of the two Nambu-Goto actions for the two membranes contains a term,

$$\int (dA - dA') \wedge \mathcal{C}^{(1)}. \quad (3.5)$$

Thus, it is the magnetic flux of the difference of the two gauge fields that generates D0-brane charge [7]. This is precisely the gauge field identified by Sen as being Higgsed by the perturbative tachyon, whose magnetic vortex generates the D0 charge. The other linear combination $A + A'$ is left intact by the perturbative sector of Type II superstrings, and its electric flux corresponds to the net winding in figures 1 and 2. This is precisely the world-volume gauge field that was argued to be confined.

In effect, we established confinement for the case of D2-anti-D2, at least when the M-theory description is appropriate. Because the above confinement of the electric flux can be seen from a geometrical viewpoint of space-time, we will call this phenomenon “geometric confinement.”

3.2 More Decay Channels of Brane-anti-Brane Pairs

While we started with the D2-anti-D2 system (motivated by the proposal of Ref. [11]), the above mechanism easily generalizes to other cases. Whenever one finds a $(p+1)$ -dimensional brane-anti-brane pair transverse to a compact circle, one can dualize the compact scalar ξ associated with the position of the p -brane to a $(p-1)$ -form tensor field,

$$\partial_i \xi = \frac{1}{(p-1)!} \epsilon_i^{j_1 \dots j_{p-1}} \mathcal{A}_{j_1 \dots j_{p-1}}. \quad (3.6)$$

Defining ξ' and its dual \mathcal{A}' analogously for the anti- p -brane, $\mathcal{A} + \mathcal{A}'$ will undergo a similar geometric confinement process. The confined “electric” flux forms a domain-wall. From the space-time perspective, the domain wall is simply a p -brane of the same kind wrapped along the compact direction. An interesting example is the case of M5-anti-M5-branes transverse to X^{11} , namely an NS5-anti-NS5-brane pair, where the resulting domain-walls are wrapped M5-branes, also known as D4-branes.

Starting with a Dp -anti- Dp pair that is transverse to a circle X^9 , the confined “electric” flux of $\mathcal{A} + \mathcal{A}'$ corresponds to a Dp -brane which is wrapped along X^9 . Actually, the compact “scalar” $\xi - \xi'$ associated with the relative position of the pair along X^9 is not

really a scalar. In the T-dual picture, it is part of the gauge field, where the phenomenon is annihilation of a $D(p+1)$ -anti- $D(p+1)$ pair through condensation of the open string tachyon. The T-dual of the wrapped Dp -brane is a transverse $D(p-1)$ -brane, which is realized as a magnetic vortex on the longitudinal $D(p+1)$ -anti- $D(p+1)$ world-volume. Only when the compact direction R_9 is large compared to the string scale, we may consider ξ as a scalar field. In that limit, the dual circle along \tilde{X}^9 has a small radius $1/R_9$, and a vortex on $D(p+1)$ -anti- $D(p+1)$ world-volume is de-localized along this compact direction, and effectively becomes a domain-wall configuration. Thus, one may regard geometric confinement on a Dp -anti- Dp pair as a special limit of Sen's tachyon condensation, seen from a T-dual perspective.

Finally, we also believe that there is a mechanism which is complementary to geometric confinement. It is only one linear combination of $\mathcal{A} + \mathcal{A}'$ that is confined by the latter. The other combination $\mathcal{A} - \mathcal{A}'$ must acquire a mass-gap by some other means as well. In the case of Dp -anti- Dp transverse to X^9 , this mechanism would be T-dual to confinement of the vector field already proposed in Ref. [11]. Since the fundamental string wrapped along X^9 transforms into a KK momentum mode along X^9 , the solitonic by-product one obtains is a Kaluza-Klein momentum mode. Analogously, we expect the possible solitonic by-products of the $M5$ -anti- $M5$ system transverse to X^{11} to include a $D0$ -brane.¹ (In another special case of $M2$ -anti- $M2$ pairs transverse to compact X^{11} , the mechanism is again nothing but Sen's open string tachyon condensation, whose solitonic by-product is a $D0$ -brane.)

This, together with the ordinary confinement mechanism of Ref. [11], suggests to us an interesting possibility. When a D-brane and an anti-D-brane annihilate in the presence of compact circles, certain perturbative closed string states could emerge. The states that we have found are all BPS (they carry either a winding number or a KK momentum of fundamental strings) and thus relatively easy to probe. Taking this one step further, one may be persuaded that the entire closed string spectrum should be reproduced from dynamics of open strings.

4 Confinement at Weak String Coupling

Since the world-volume gauge theory description of the D-brane dynamics is valid only at weak string coupling $g_s \ll 1$, it is clear that the issue of the unbroken $U(1)$ should be

¹Ref. [18] identified the couplings between the space-time and the world-volume fields which are necessary for these additional decay modes of $M5$ -anti- $M5$.

addressed in this regime. The main question concerning the confinement mechanism as a possible resolution is that it involves highly non-perturbative objects in string theory with a huge tension $\sim 1/g_s$. In particular, it is not clear that such objects give the most important effect. Similarly, “geometric confinement” is established only at strong Type IIA coupling, where a semi-classical description of the membranes is valid. A priori, confinement at strong coupling need not imply confinement at weak coupling.

Recently, in the study of the effective action of unstable D p -branes in Type II string theory, Sen proposed a new mechanism to explain the disappearance of the unbroken $U(1)$ purely in terms of tree-level string theory [12]. Using the results of [19] on brane dynamics in the background of a constant magnetic or B field, it was shown that the bosonic part of the effective action in terms of the $U(1)$ gauge field strength $F = dA_+$ and the tachyon field T , is given by

$$S = -\frac{1}{g_s} \int d^{p+1}x \sqrt{-\det(g + F)} V(T), \quad (4.1)$$

under the assumption that F and T are constant. In the above expression $V(T)$ is the tachyon potential and g is the induced metric on the world-volume, which is also assumed to be constant. This can be extended to the system of D-brane and anti-D-brane, where F is the gauge field of the unbroken $U(1)$, under the additional assumption that the gauge field of the broken $U(1)$ vanishes (we thank A. Sen for discussion on this point).¹ Here we are assuming that the NS B -field is zero, but it is useful to keep in mind that a non-zero B -field would enter in (4.1) as $F \rightarrow F + B$. It has been argued that the tachyon potential $V(T)$ vanishes at its bottom. If this is true, the gauge kinetic term vanishes at the bottom of the tachyon potential. In [12] it was further argued that this means that the $U(1)$ gauge field acts as a Lagrange multiplier which removes all charged objects from the spectrum. Since this is a tree-level effect in string theory, it appears to be a perfect resolution to the puzzle. Is the non-perturbative confinement mechanism of no relevance for the resolution?

4.1 $p = 1$ Case

To examine the consequence of (4.1) more carefully, let us consider the simplest non-trivial example, $p = 1$. It may appear that the issue of the unbroken $U(1)$ is absent in this case, since $1 + 1$ dimensional gauge fields have no propagating degrees of freedom.

¹We are using the normalization where the two endpoints of an open fundamental string carry unit charges with respect to the gauge fields A and A' . We also set $2\pi\alpha' = 1$, so that the tension of the fundamental string is 1. Also, we take the convention that $V(T)/g_s$ at $T = 0$ is the brane tension. Thus, $V(0)$ is $\sqrt{2}(2\pi)^{\frac{1-p}{2}}$ for the unstable D p -brane and $2(2\pi)^{\frac{1-p}{2}}$ for the D p -anti-D p -brane pair.

However, there are actually topological degrees of freedom when the theory is formulated on a compact space, and these require consideration.

We consider a D1-anti-D1 pair, or an unstable D1-brane, wrapped on a circle of radius R . The effective action is given by (4.1) with $p = 1$, and the integral is over $\mathbf{R} \times S^1$. We focus on the dynamics of the gauge field A , and fix all other degrees of freedom. In particular, we assume that the induced metric g is the diagonal matrix $g_{11} = -g_{00} = 1$, $g_{01} = 0$. The system is then described by a single bosonic gauge invariant variable a of period $1/R$, which parameterizes the holonomy on S^1 as $\int_{S^1} A = 2\pi R a$. The world-volume theory is now reduced to the quantum mechanics obtained from the Lagrangian

$$L = -\frac{2\pi R}{\lambda} \sqrt{1 - \dot{a}^2}, \quad (4.2)$$

where

$$\frac{1}{\lambda} = \frac{1}{g_s} V(T). \quad (4.3)$$

This is identical to the Lagrangian for the free relativistic particle of mass $2\pi R/\lambda$. Thus, the Hamiltonian is given by

$$H = \sqrt{p^2 + (2\pi R/\lambda)^2}, \quad (4.4)$$

where p is the momentum conjugate to a , $p = \partial L/\partial \dot{a} = (R/\lambda)\dot{a}/\sqrt{1 - \dot{a}^2}$. Since a is a periodic variable of period $1/R$, the wave functions are spanned by $\psi_n(a) = e^{2\pi i R n a}$, for integer n . These are eigenstates of the momentum $p = -i\partial/\partial a$ with eigenvalues $2\pi R n$. The energy of the state ψ_n is given by

$$E_n = 2\pi R \sqrt{n^2 + \frac{1}{\lambda^2}}. \quad (4.5)$$

The ground state is ψ_0 and has energy

$$E_0 = R \times \frac{1}{g_s} V(T). \quad (4.6)$$

At the bottom of the tachyon potential, where we assume $V(T) = 0$, the ground state has a vanishing energy, and can be identified with the supersymmetric vacuum of the string theory. Conversely, the identification of the ground state with the supersymmetric string vacuum requires the tachyon potential $V(T)$ to vanish at its bottom.

Now consider the excited states ψ_n with $n \neq 0$. In the limit of vanishing $V(T)$, ψ_n has a finite energy

$$E_n \longrightarrow 2\pi R |n|, \quad (4.7)$$

and remains in the spectrum. Note that the momentum $p = \partial L/\partial \dot{a}$ is also a source for the Neveu-Schwarz B-field B_{01} [2]. Thus ψ_n must represent a state in string theory with

the fundamental string wrapped n times on S^1 . Indeed, the energy (4.7) is precisely the mass of an n -wound fundamental string. So the electric flux “tube”, which in this case fills the $1 + 1$ dimensional world-volume, can truly be identified with the fundamental string.

In the present discussion, we have implicitly assumed that (4.1) holds for arbitrary configurations of the field, and ignored the probable higher derivative corrections. To be very precise this requires justification, even though all the eigenstates have constant field strengths in the present case.

4.2 $p \geq 2$ Cases

The above example explicitly shows that Sen’s result (4.1) together with the assumption that $V(T) = 0$ at the bottom does not really eliminate the unbroken $U(1)$ degrees of freedom, but rather supports the idea of confinement. The key point was that a massless relativistic particle can carry a non-zero energy. Even though the Lagrangian appears to vanish in the massless limit, the Hamiltonian remains non-trivial. We expect a similar and possibly more interesting story in the $p > 1$ cases.

Actually, an important aspect was ignored in the argument for the resolution of the puzzle as a direct consequence of (4.1). The vanishing of the gauge kinetic term is equivalent to the blow-up of the gauge coupling. When the gauge coupling becomes large, one has to start worrying about strong gauge dynamics, and it is usually impossible to describe this in the original variable. In other words, the description in terms of the gauge field does not really make sense when the tachyon expectation value comes close to the bottom of the potential.

When one description breaks down, one must go over to another, better, description. In the theory of a 1-form gauge potential in $(p + 1)$ -dimensions with a large coupling, the better description is in terms of the dual $(p - 2)$ -form potential with inverse coupling. The latter becomes better as the original coupling becomes larger. Now, it is clear that of utmost relevance is the object of least mass or tension that is charged under the $(p - 2)$ -form potential. For $p \geq 3$, it is precisely the $(p - 3)$ -brane in the world-volume coming from the stretched $D(p - 2)$ -brane. It is therefore tempting to consider confinement of the unbroken $U(1)$ by the magnetic Higgs mechanism as the actual resolution of the puzzle. For $p = 2$, this does not work since the “ $(p - 3)$ -dimensional object” does not exist as a charged object. However, Euclidean D0-branes can stretch between D2-branes, and can modify the dynamics by an instanton effect. We shall first consider this case in detail, postponing the $p \geq 3$ cases for later discussion. Throughout the discussion we assume

that higher derivative corrections to (4.1) can be ignored.

$$p = 2$$

It was shown by Polyakov in [20] that a $U(1)$ gauge theory in 2+1 dimensions which includes monopole instantons exhibits confinement by the instanton effect of the monopoles. This is quite similar to the situation under consideration. However, there are also notable differences — we are considering the Born-Infeld action rather than the standard Maxwell action, and we do not know the size of the monopole-instanton (“W-boson mass”) or the value of the instanton action. In particular, it appears hopeless to compute the tension of the confined electric flux tube, even if we could argue for confinement. Nevertheless, under a certain assumption on the instanton effect, we can argue for confinement and even compute the exact tension of the flux tube.

Let us first dualize the $U(1)$ gauge field ignoring the effect of D-brane instantons.¹ We start with a system of a $U(1)$ gauge field A and a one-form field Π with the action

$$S' = -\frac{1}{\lambda} \int d^3x \sqrt{-g} \sqrt{1 + \lambda^2 |\Pi|^2} + \int \Pi \wedge F, \quad (4.8)$$

where $F = dA$ is the curvature of A , and $|\Pi|^2 = g^{\mu\nu} \Pi_\mu \Pi_\nu$. We first integrate over the one-form field Π in the stationary phase approximation. The action S' is stationary at

$$\Pi = \frac{1}{\lambda} \frac{*F}{\sqrt{1 - |F|^2}}, \quad (4.9)$$

where $*$ is the Hodge dual with respect to the metric $g_{\mu\nu}$, and $|F|^2 = \frac{1}{2} g^{\mu\nu} g^{\rho\kappa} F_{\mu\rho} F_{\nu\kappa}$. Inserting this value into (4.8) we obtain the action for A

$$S = -\frac{1}{\lambda} \int d^3x \sqrt{-g} \sqrt{1 - |F|^2}. \quad (4.10)$$

It is easy to see that this is equal to (4.1) with $p = 2$, if λ is given by

$$\frac{1}{\lambda} = \frac{1}{g_s} V(T). \quad (4.11)$$

Next, let us exchange the order of integration. Integrating out the gauge field A , we obtain a constraint that Π is a closed one form with integral period on one-cycles. In other words, Π can be written as

$$\Pi = d\sigma, \quad (4.12)$$

¹See Ref. [21] for related discussions on dualization of Born-Infeld action.

where σ is a periodic variable of period 1 (so that $e^{2\pi i\sigma}$ is a circle valued function). Now the action in terms of σ is

$$\tilde{S} = -\frac{1}{\lambda} \int d^3x \sqrt{-g} \sqrt{1 + \lambda^2 |d\sigma|^2}. \quad (4.13)$$

If λ were just the string coupling this would of course be the same as the action of the wrapped membrane in M-theory on $\mathbf{R}^{10} \times S^1$ written in string units. Let us consider the limit $\lambda \rightarrow \infty$ corresponding to $V(T) \rightarrow 0$, but still with $g_s \ll 1$. Then the dual action (4.13) has a finite limit

$$\tilde{S} \rightarrow - \int d^3x \sqrt{-g} |d\sigma| = - \int d^3x \sqrt{-g} \sqrt{g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma}. \quad (4.14)$$

This is completely different from the membrane action in \mathbf{R}^{11} , which one would obtain from the dual of the D2-brane action by sending $g_s \rightarrow \infty$, keeping finite the eleven-dimensional Planck length. The classical energy density of a static configuration is given by

$$\mathcal{E} = \sqrt{g^{ik} \partial_i \sigma \partial_k \sigma}, \quad (4.15)$$

where i, k are spatial coordinate indices. In particular, the ground state has zero energy.

Now let us turn on an electric flux F_{01} in the x^1 -direction of the world-volume. Comparison of (4.9) and (4.12) shows that this corresponds to turning on $\partial_2 \sigma$. Namely,

$$\Delta\sigma := \sigma(x^2 = +\infty) - \sigma(x^2 = -\infty) = \int_{-\infty}^{+\infty} \frac{1}{\lambda} \frac{F_{01} dx^2}{\sqrt{1 - |F|^2}}. \quad (4.16)$$

This quantity can be identified as the charge of fundamental strings stretched in the x^1 -direction, since the integrand of the right hand side is the same as $\delta S / \delta F_{01} = \delta S / \delta B_{01}$. The flux carries energy density along the x^1 -direction given by

$$T = \int dx^2 |\partial_2 \sigma| = |\Delta\sigma|, \quad (4.17)$$

if σ is a monotonic function. Since $T = 1$ for a unit charge $|\Delta\sigma| = 1$, it may appear that the flux can really be identified as the fundamental string. However, we note that (4.17) holds for *any* monotonic function σ , which can spread out without changing the tension. Thus, the electric flux does not tend to squeeze into a thin tube, and confinement does not appear to occur. Furthermore, even if we force the flux to be confined in a tube, the charge $\Delta\sigma$ is not quantized (unless the x^2 -direction is compact) and we cannot truly identify the tube as the fundamental string.

This can be cured by taking into account the effect of the instantons. For the D2-anti-D2 pair in Type IIA string theory, a Euclidean trajectory of a D0-brane plays the

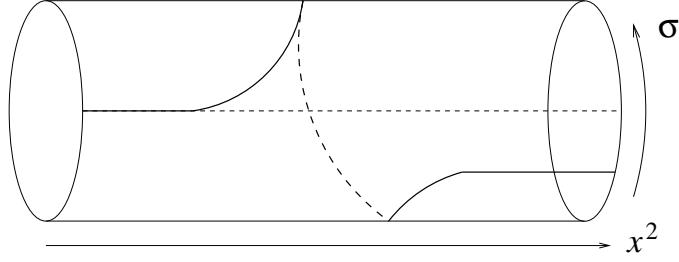


Figure 3: *Any* configuration attains minimum energy density (4.17) as long as $\sigma(x^2)$ is a monotonic function. Furthermore, the charge $\Delta\sigma$ is not quantized.

role of the instanton, whereas for the Type IIB unstable D2-brane, the $(-1)^{F_L}$ projection of the Euclidean D0-anti-D0 pair will do the job. We recall that the instanton creates a point defect at x in the 3-dimensional world-volume, from which a unit of magnetic flux emanates. On a small sphere S^2 surrounding x we have $\int_{S^2} F/2\pi = 1$ (it would be -1 for an anti-instanton). In the magnetic description in terms of σ , this corresponds to an insertion of the operator $e^{2\pi i\sigma(x)}$ in the path-integral ($e^{-2\pi i\sigma(x)}$ for anti-instanton). This can be seen as follows (this presentation is basically from [22]). Insert the operators $e^{2\pi i\sigma(x)}$ and $e^{-2\pi i\sigma(y)}$ at distinct points $x \neq y$. Because of (4.12), in the original description (4.8) this corresponds to the insertion of

$$\exp\left(2\pi i \int_y^x \Pi\right), \quad (4.18)$$

where the integral in the exponent is over any path starting at y and ending at x . This integral can be written as an integral over the 3-dimensional world-volume X as

$$\int_y^x \Pi = \int_X \Pi \wedge \omega, \quad (4.19)$$

where ω is a closed two form with δ -function support on the path, so that $\int_H \omega = 1$ for any oriented hyperplane H intersecting once with the path. For example, ω has period 1 and -1 on small 2-spheres S_x^2 and S_y^2 surrounding x and y , respectively. Then the term $\int \Pi \wedge F$ in (4.8) is replaced by $\int \Pi \wedge (F + 2\pi\omega)$, and the action after integration over Π is given by (4.10) with F replaced by $F' = F + 2\pi\omega$. Now, F' satisfies

$$\int_{S_x^2} F'/2\pi = - \int_{S_y^2} F'/2\pi = 1, \quad (4.20)$$

and is therefore a curvature of a gauge field on $X - \{x, y\}$ which has a unit magnetic flux emanating from x and going into y . This shows the claim.

Summing over a gas of instantons and anti-instantons corresponds to the insertion of

$$\begin{aligned} \sum_{n_+, n_-} \frac{1}{n_+! n_-!} \int \prod_{i=1}^{n_+} d^3 x_i \mu^3 e^{2\pi i \sigma(x_i)} \prod_{j=1}^{n_-} d^3 y_j \mu^3 e^{-2\pi i \sigma(y_j)} \\ = \exp \left(\mu^3 \int d^3 x (e^{2\pi i \sigma(x)} + e^{-2\pi i \sigma(x)}) \right) \end{aligned} \quad (4.21)$$

in the Euclidean path-integral, where μ is some mass parameter which we assume to be non-zero. The instanton effect therefore generates a potential energy proportional to $\cos(2\pi\sigma)$, and the new effective action in the limit $\lambda \rightarrow \infty$ becomes

$$\tilde{S}_{\text{eff}} = - \int d^3 x \sqrt{-g} (\sqrt{g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma} + M^3 \cos(2\pi\sigma)), \quad (4.22)$$

where M is the $\lambda \rightarrow \infty$ limit of μ , which is again assumed to be non-zero. The energy density of a static configuration is then given by

$$\mathcal{E}_{\text{eff}} = \sqrt{g^{ik} \partial_i \sigma \partial_k \sigma} + M^3 (\cos(2\pi\sigma) + 1), \quad (4.23)$$

where we have added a constant so that the system attains zero energy in the ground state. We do not know how to generate this energy shift at the moment, but we take it for granted since the non-negativity of energy is required on general grounds.

The formula (4.17) for the energy density of the flux is thus modified to

$$T = \int dx^2 (|\partial_2 \sigma| + \sqrt{g_{22}} M^3 (\cos(2\pi\sigma) + 1)). \quad (4.24)$$

In order for the tension to be finite, σ must approach vacuum values as $x^2 \rightarrow \pm\infty$. The charge is therefore quantized,

$$\Delta\sigma = n \in \mathbf{Z}. \quad (4.25)$$

Also, the potential term prevents the flux from spreading, and confines it into a thin flux tube. In the limit of a completely squeezed configuration, the tension is equal to the flux,

$$T \rightarrow |n|. \quad (4.26)$$

This is because the contribution from the potential vanishes in this limit. In particular, the result is independent of the value of M , which is difficult to estimate without a detailed knowledge of the instanton. This is the magic of the limit $\lambda \rightarrow \infty$; if σ had a standard kinetic term $|\partial\sigma|^2$, the stable configuration would have been determined by a non-trivial balance between two effects — spreading by the kinetic term and squeezing by the potential term. In our case, the spreading effect of the kinetic term vanishes in the limit $\lambda \rightarrow \infty$.

To summarize, we have seen that due to the effect of the D-brane instantons, the electric flux is squeezed into a thin flux tube and is quantized. The flux tube has the correct charge and tension to be identified with the fundamental strings.

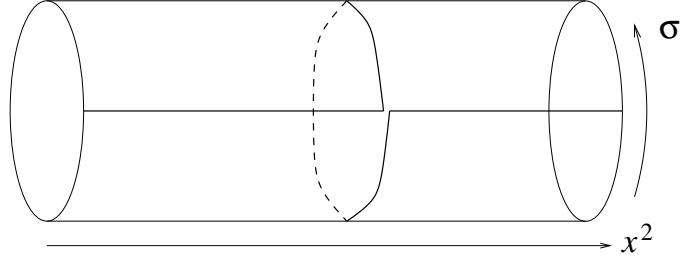


Figure 4: By the instanton effect, the charge $|\Delta\sigma|$ is quantized and the energy density (4.24) is minimized for the completely squeezed configuration. This density, or the tension of the resulting string, is proportional to the charge.

$$p \geq 3$$

We finally discuss the $p \geq 3$ cases. We do not attempt to make a quantitative analysis here. However, we provide a possible picture of the physics in the weak string coupling regime $g_s \ll 1$ (See Figure 5). To be specific, we consider a Dp -anti- Dp system, but the generalization to unstable Dp -branes is obvious. (We ignore the factors of $(2\pi)^{\frac{1-p}{2}}$ in this discussion.)

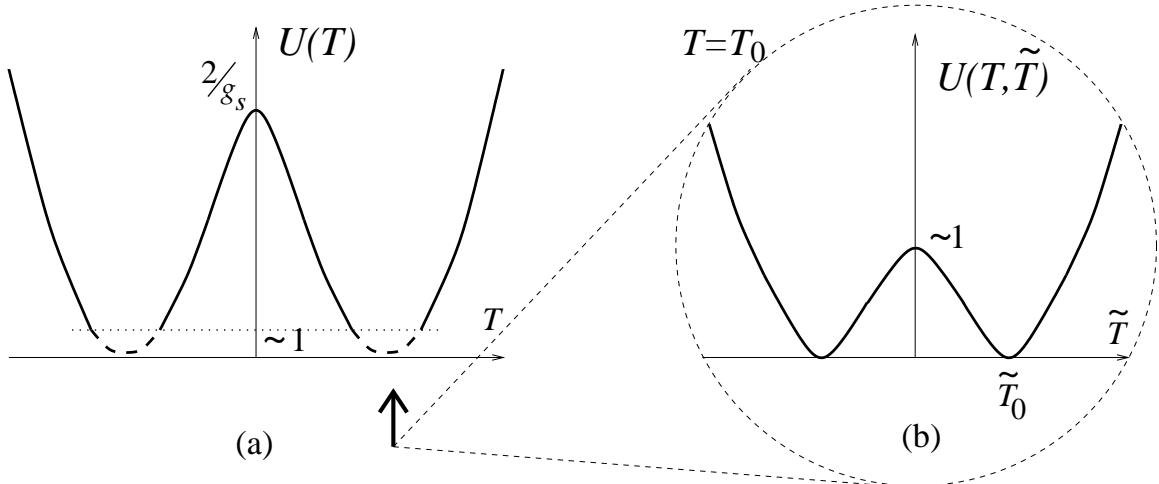


Figure 5: The tachyon potential (a), and the tachyon-magnetic-tachyon potential at the vacuum value of $T = T_0$ (b).

- The system at $T = 0$ has energy $2/g_s$. Here both $U(1)$ gauge groups are unbroken,

and the corresponding gauge fields A_- and A_+ are weakly coupled. In particular the Lagrangian for $U(1)_+$ is given by (4.1) with $V(T) = 2$. This point is unstable and the tachyon T tends to acquire non-zero values.

- Once the tachyon condenses $T \neq 0$, $U(1)_-$ is broken by the Higgs mechanism, and its gauge boson becomes massive. $U(1)_+$ remains unbroken. The potential energy $U(T) = V(T)/g_s$ decreases as T rolls down toward its vacuum value T_0 . But as long as $U(T)$ is much larger than 1, the $U(1)_+$ gauge coupling is small, and one can still use the gauge theory description of (4.1). Of course the tachyon continues to roll down to smaller values of $U(T)$.
- When $U(T)$ comes close to 1, the description in terms of the gauge field A_+ is no longer appropriate, and one should use the dual magnetic description. Standard electric-magnetic duality turns the gauge field A_+ to a $(p-2)$ -form gauge potential \tilde{A}_+ .
- A $(p-3)$ -dimensional object charged under \tilde{A}_+ is created by the stretched $D(p-2)$ -brane, and we denote the corresponding “field” by \tilde{T} . Thus the system in this region is described by the fields (T, A_-) and (\tilde{T}, \tilde{A}_+) , which are possibly coupled by a potential $U(T, \tilde{T})$. Since we have dualized at $U(T) \sim 1$, we expect the potential energy at $\tilde{T} = 0$ to be of order 1. The dynamics is hard to analyze, but we assume that \tilde{T} is tachyonic at $\tilde{T} = 0$ in constant- T hyperplanes, and tends to acquire non-zero expectation values (See Figure 5 (b)).
- Once the magnetic tachyon condenses $\tilde{T} \neq 0$, the magnetic $U(1)_+$ is broken by the Higgs mechanism, and the gauge boson \tilde{A}_+ becomes massive. In other words, the original $U(1)_+$ is confined. We assume that the potential $U(T, \tilde{T})$ vanishes at its bottom ((T_0, \tilde{T}_0) in Figure 5 (b)), and has positive second derivatives there. The vacuum configuration is then indistinguishable from the supersymmetric vacuum of Type II string theory.

We recall that a vortex configuration of the tachyon T with the gauge field A_- is identified with the BPS $D(p-2)$ -brane [7]. This is not altered in the new picture. One must ensure however that both T and \tilde{T} attain their vacuum expectation values away from the core of the vortex. The contribution to the energy from the (\tilde{T}, \tilde{A}_+) sector, even if exists, is of order 1 and is negligible compared to the contribution $\sim 1/g_s$ from the (T, A_-) sector. Thus, the vortex can have the correct tension $1/g_s$ to be identified as the BPS $D(p-2)$ -brane.

There is another topological defect which comes purely from the (\tilde{T}, \tilde{A}_+) sector, with T at its vacuum value everywhere in the world-volume. This is a topologically non-trivial configuration such that \tilde{A}_+ has a quantized period over a $(p-2)$ -dimensional sphere surrounding a one-dimensional object in the world-volume. This one-dimensional object can be identified as the confined electric flux tube of $U(1)_+$ via electric-magnetic duality. This in turn is identified with the fundamental string, as follows from the coupling of $F = dA_+$ to the NS B -field. Indeed, since the typical energy scale of the (\tilde{T}, \tilde{A}_+) system is of order 1, once the open string tachyon rolls down to its vacuum value T_0 , the topological defect has tension of order 1 (the point is that the topological defect requires \tilde{T} to deviate from the vacuum but T can remain at T_0). This is the correct value of the tension for the fundamental string.

4.3 The Fate of Massless Scalars

So far, we have focussed on the world-volume gauge fields. However, for $p < 9$, the issue of the massless gauge boson for the unbroken $U(1)$ is actually only a part of the problem; there are also $(9-p)$ massless scalars, which represent the center of mass motion of the D p -anti-D p pair or the unstable D p -brane.¹ Thus, in order to really solve the problem, we must clarify the fate of these massless scalars. We will not attempt to fully solve this problem. Instead, we examine the situation in the simplest case of $p = 0$, and briefly comment on the case of $p \geq 1$.

Unlike for gauge bosons, the puzzle of the massless scalars exists already for $p = 0$. To be specific, let us consider a D0-anti-D0 pair in Type IIA string theory on \mathbf{R}^{10} . After tachyon condensation, the nine scalars representing the relative motion become massive, while the other nine scalars ϕ^i ($i = 1, \dots, 9$) representing the center of mass motion remain massless. The Lagrangian for the massless fields is

$$L = -\frac{1}{\lambda} \sqrt{1 - \sum_{i=1}^9 (\dot{\phi}^i)^2}. \quad (4.27)$$

This describes a relativistic particle in \mathbf{R}^{10} of mass $1/\lambda = V(T)/g_s$. Its mass therefore vanishes at the bottom of the tachyon potential, where it is assumed that $V(T) = 0$. In other words, *the fate of the center of mass scalar fields in the D0-anti-D0 pair is to produce a massless particle in \mathbf{R}^{10}* . It is natural to interpret the latter as a massless particle in the closed string spectrum. In fact, if the D0-anti-D0 pair annihilates and no open string mode remains, this is the only possible interpretation.

¹In the case of D p -anti-D p pair, the relative motion is frozen by tachyon condensation, which gives mass to the corresponding scalars.

The fate of the center of mass scalars is less clear for $p \geq 1$. However, they cannot be ignored altogether simply because the Lagrangian vanishes at the bottom of the tachyon potential. To see this, let us consider the situation where an electric flux of the gauge theory sector is turned on and is confined into a thin tube. The action of the system can be factorized as

$$S = - \int_{\mathbf{R} \times W} d^{p+1}x \sqrt{-\det g} \frac{1}{\lambda} \sqrt{\det(1 + g^{-1}F)}, \quad (4.28)$$

where W is the spatial part of the $p+1$ dimensional world-volume. Confinement of electric flux in the gauge sector means that one can replace the factor $\frac{1}{\lambda} \sqrt{\det(1 + g^{-1}F)}$ by a delta function on W supported along the one-dimensional flux string $C \subset W$. The action can then be expressed as

$$S = - \int_{\mathbf{R} \times C} d^2\sigma \sqrt{-\det \gamma}, \quad (4.29)$$

where γ is the metric induced on the flux string, which is determined by the massless scalar fields (restricted on $\mathbf{R} \times C$). Thus, the massless scalar fields restricted to the flux string represent the motion of the flux string in directions transverse to the world-volume of the Dp -anti- Dp system. These “new” degrees of freedom² are actually required in order to identify the flux string as the fundamental string, since the fundamental string can move in the full eight transverse dimensions. Without this, the string would have been trapped in a plane W of co-dimension $(9 - p)$.

While this example shows that the scalar degrees of freedom play an important role, it stops short of explaining fully how the $(p + 1)$ -dimensional scalar fields really reduce to 2-dimensional ones. Their action vanishes outside the string core, but as we have seen earlier, a vanishing action does not automatically imply disappearance of the associated degrees of freedom.

5 Conclusions

We started with Sen’s simple observation that brane-anti-brane annihilation should lead to the supersymmetric vacuum of closed string theory, and tried to understand how (some of) the open string degrees of freedom are removed from the low energy spectrum. The basic mechanisms that lift the massless gauge sectors of the lowest lying open string modes have been identified as the perturbative Higgs effect [7] and non-perturbative confinement [11]. We have shown that the two effects are in fact linked: the former process forces us, via Sen’s effective action, to describe the unbroken gauge sector using the dual

²It is new compared to the motion of the string within the brane W . The latter should emerge as Goldstone bosons associated with translation along W .

non-perturbative degrees of freedom, which naturally allow us to describe confinement of the unbroken $U(1)$ as a weakly coupled dual Higgs mechanism. The combined effect is to lift all gauge sectors in the lowest lying open string modes. A careful consideration along similar lines might reveal how the remaining massless scalar fields are lifted as well.

More generally, we believe that confinement is one of the central ingredients in converting open string degrees of freedom to closed string ones. To see this, imagine that we have semi-classical open strings suspended between a pair of D-branes. Consider two such strings of opposite orientations, which are well separated. On the world-volume of each D-brane, a unit of electric flux emanates from the end of one string and converges on that of the other. Introducing an anti-D-brane to annihilate against one of the D-branes, we should find that no string remains ending on the annihilated D-brane. How is this accomplished via local interactions on the branes? The obvious answer is that the electric flux gets confined, and becomes a segment of fundamental string that connects the two ends of the semi-classical open strings. We can also consider an analogous process where a semi-classical open string with both ends on a D-brane is converted to a semi-classical closed string, by annihilation of the D-brane against another. This picture is quite suggestive.

In a sense, one of the most surprising aspects of brane-anti-brane annihilation is that the process can be described, at least partly, from the world-volume perspective. To understand the annihilation process more completely, one possible approach would be to solve the corresponding open string field theory [12, 13]. At the moment, it is not clear to what extent such a program can be carried out, especially in the face of the non-perturbative processes we encountered.

We have found the importance of the result (4.1) of [12] in the world-volume field theory approach. To be precise, this result is exact only for a constant field strength and a constant induced metric, and higher derivative corrections are probable. We have assumed that such corrections can be ignored in our argument for confinement, but of course this requires a considerable justification. It is hoped that our result provides a strong motivation to clarify and estimate the validity of (4.1) using various methods, including open string field theory.

Other immediate problems include generalizing our observations to other cases, such as D5-anti-D5 annihilation in Type I theory. One can imagine that an open D3-brane ending on the D5's would play a role¹, but a further difficulty arises from the fact that the would-be-confined gauge sector is of $Sp(1)$. We hardly understand what it means to

¹Absence of a closed D3 in Type I theory does not exclude the possibility of an open D3 ending on D5's, in much the same way that the absence of a closed string does not imply the absence of open strings.

have a dual Higgs mechanism for $Sp(1)$. Another interesting question is whether there is analog of K-theory [10] for the confined gauge sector. It would be classified by “K-theory” of geometrical objects associated with the antisymmetric tensor field (possibly “gerbes”) dual to the gauge field.

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